ルーペの倍率計算法

国立身体障害者リハビリテーションセンター学院
視覚障害学科　小林　章

1　一般の倍率式の意味

$$M = D/4$$

M：倍率　D：レンズ度数（dipolar）　「4」とは何か？

レンズの倍率は Relative Distance Magnification（相対的距離による拡大）により求められる。

ある物体を見たときの網膜像の大きさ（or 視角）は視距離に反比例する。

例　40cm の距離で 10pt の文字を読んでいた人が、視距離を 20cm に短縮した場合、網膜像は 2 倍の 20pt 相当の大きさになる。

レンズの倍率を求めるには、裸眼で見たときの視距離とレンズを通して見たときの視距離の比を使っている。裸眼で見たときの視距離を仮に 25cm（４D 相当）と設定し、レンズを使ったときの視距離と比較している。つまり、$D/4$ の４は４D の意味であり、通常の人は４D の調節力を持っていることにしているのである。例えば、４D の調節力を持つ人が８D のレンズを使って物体を見た場合、裸眼の最短視距離は 25cm、レンズを使った視距離が 12.5cm で倍率は $\frac{1/0.125}{1/0.25} = 2$ となる。　したがって $M = D/4$ の計算式は万能の式ではなく、調節力が 4D 以上ある人にしか使えないものである。調節力が３D の人は $M = D/3$　２D の人は $M = D/2$　となる。つまり、同じ度数のレンズを使っても、調節力が低いほど相対的なレンズの倍率が高くなるといえる。

2　物体をレンズの焦点距離上に置いたときの倍率（Maximum Angular Magnification）

（以下出典 Rodney W. Nowakowski, PRIMARY LOW VISION CARE, APPLETON & LANGE, 1994）

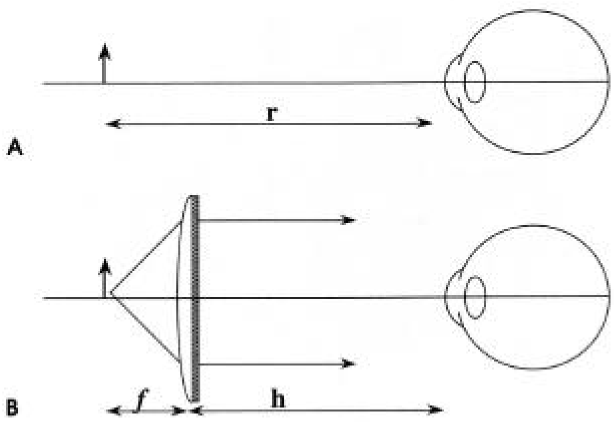


Figure 10–5. A convex lens held one focal length from the object of regard will create its maximum angular magnification, which is the total magnification to the viewer if the object remains at the reference distance. A clinical example would be as follows: **(A)** The patient reads an acuity card at some reference distance r, and then, **(B)** without moving the print, he or she places a convex lens one focal length in front of it. The distance of the lens from the eye is h; therefore, r = *f* + h, assuming the lens to be thin.

左図のように見たときの最大倍率は

$$M = 1 + hF$$

となる。

F：レンズ度数

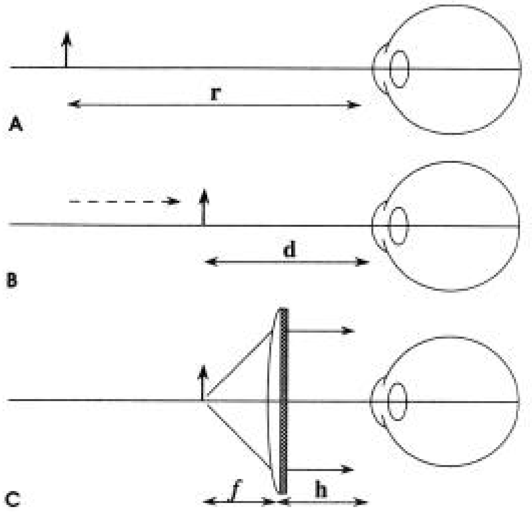


Figure 10–6. If an object is moved to a distance other than the reference distance, the viewer will receive the maximum angular magnification from the lens if it is held one focal length from the object, and relative distance magnification from the position change. A clinical situation would be as follows: **(A)** The patient reads an acuity card at some reference distance r, moves the print **(B)** to some new distance d, and then places a convex lens one focal length in front of the print **(C)** at the new distance. The distance of the lens from the eye is h; therefore, d = *f* + h, if the lens is considered to be thin.

レンズによる拡大と相対距離による拡大をあわせて利用すると次のような計算式が導かれる。

$$M_{total} = RDM \times MAM$$

$$M_{total} = r / d \times (1 + hF)$$

$$= r/(f+h) \times (1 + hF)$$

$$= r/(f+h) \times (f/f + hF)$$

$$= r/(f+h) \times (f/f + h/f)$$

$$= r/(f+h) \times (f+h)/f$$

$$= r/f$$

$$= rF$$

例題をやってみましょう！

■ **Example 10–7.** A patient reads 4M print at a distance of 40 cm through a +2.50 add over the best correction. What size print could she read with a +5.00 D hand-held lens held 20 cm from the print?

$$M = rF$$
$$M = (0.4)(5)$$
$$M = 2X$$

Since the lens provides 2X, she should now be able to read 2M letters through the distance correction, with the print held 20 cm from the lens:

$$\text{Magnification} = \text{Reference size/Goal size}$$
$$2X = 4M/\text{Goal size}$$
$$4/2 = \text{Goal size}$$
$$2 = \text{Goal size}$$

■ **Example 10–8.** A patient reads 6M print at 33 cm with a +3.00 add over the best correction. What microscope would be required to read 1M print?

$$\text{Magnification required} = \text{Reference size/Goal size}$$
$$M = 6M/1M$$
$$M = 6X$$

Once the required magnification is known, the lens power that will provide it is calculated as follows:

$$M = rF$$
$$6 = (0.33)F$$
$$6/0.33 = F$$
$$+18.18 = F$$

■ **Example 10–9.** A patient has a reference acuity of 0.4/10M through best correction and the appropriate add. The goal is to read 1M print. It requires 10X (10M/1M = 10X) to achieve that goal. The lens needed is determined as follows:

$$M = rF$$
$$10 = (0.4)F$$
$$+25 = F$$

If this patient wears a +25.00 D add over best distance correction and moves the print to 4 cm from the lens, he should read 1M print. In other words, the +25.00 diopter lens provides him with 10X magnification. Yet a manufacturer who used a reference distance of 25 cm would have labeled that +25.00 lens as a 6.25X microscope:

$$M = rF$$
$$M = (0.25)25$$
$$M = 6.25X$$

■ **Example 10–10.** Another patient has a reference acuity of 0.25/10M. (She can read the same size print as the first but must hold it almost twice as close in order to do so.) How much magnification this patient would get from that same +25.00 D microscope and the size print she would be able to read is determined as follows:

Magnification

$$M = rF$$
$$M = 0.25(25)$$
$$M = 6.25X$$

Print size

Magnification = Reference size/Goal size
Goal size = Reference size/Magnification
Goal size = 10M/6.25X
Goal size = 1.6M

In this case, only 6.25X is received from a +25.00 D lens, and she will only read 1.6M print. In order to read 1M print, this patient also needs 10X, just as in Example 10-9, but needs a +40.00 D lens to achieve it. That is determined as follows:

$$M = rF$$
$$10 = (0.25)F$$
$$+40 = F$$

This patient d

TABLE 10–1. Dioptic power, determined by M = rF, needed to provide a given magnification for selected reference distances.

	Reference Distance (Meters)								
Mag	*0.10*	*0.15*	*0.20*	*0.25*	*0.30*	*0.35*	*0.40*	*0.45*	*0.50*
2X	**20.00**	13.33	10.00	8.00	6.67	5.71	5.00	4.44	4.00
3X	30.00	**20.00**	15.00	12.00	10.00	8.57	7.50	6.67	6.00
4X	40.00	26.67	**20.00**	16.00	13.33	11.43	10.00	8.89	8.00
5X	50.00	33.33	25.00	**20.00**	16.67	14.29	12.50	11.11	10.00
6X	60.00	40.00	30.00	24.00	**20.00**	17.14	15.00	13.33	12.00
7X	70.00	46.67	35.00	28.00	23.33	**20.00**	17.50	15.56	14.00
8X	80.00	53.33	40.00	32.00	26.67	22.86	**20.00**	17.78	16.00
9X	90.00	60.00	45.00	36.00	30.00	25.71	22.50	**20.00**	18.00
10X	100.00	66.67	50.00	40.00	33.33	28.57	25.00	22.22	**20.00**

Example: A +20 lens provides 3X for a reference distance of 15 cm and 10X for a reference distance of 50 cm.

3　物体がレンズの焦点上にない場合の倍率

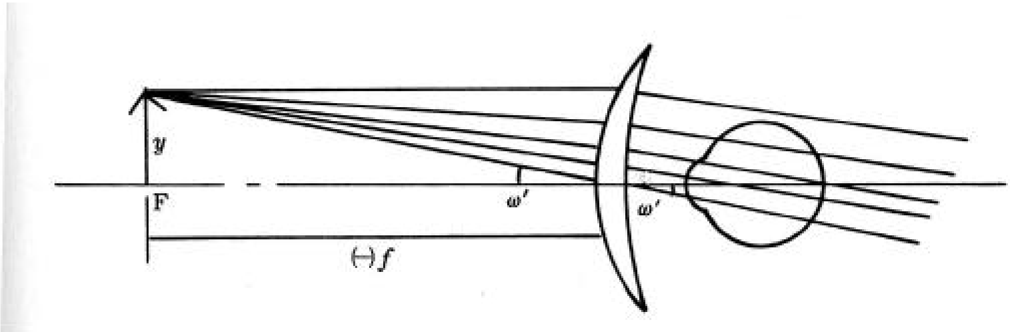
ハンドルーペを紙面から焦点距離の位置に維持することは、通常困難をともなう。紙面からレンズまでの距離、眼とレンズの距離が移動したときの倍率は、次の計算式により求められる。

$$M = \frac{rF}{1-[xF(1-hF)]}$$

r：読書距離　F：レンズ度数　χ：物体から焦点までの距離　h：眼からレンズまでの距離

4　スタンドルーペの倍率

スタンドルーペを使う際にもっとも問題になることは**「ルーペの脚の長さ≠レンズの焦点距離」**である。



「東條四郎，眼鏡光学（幾何光学的基礎），日本眼鏡専門学校」85 頁

物体が焦点上にあれば、レンズを通過した光線は平行になり、眼の位置がどこにあっても、網膜像の大きさは変わらない。

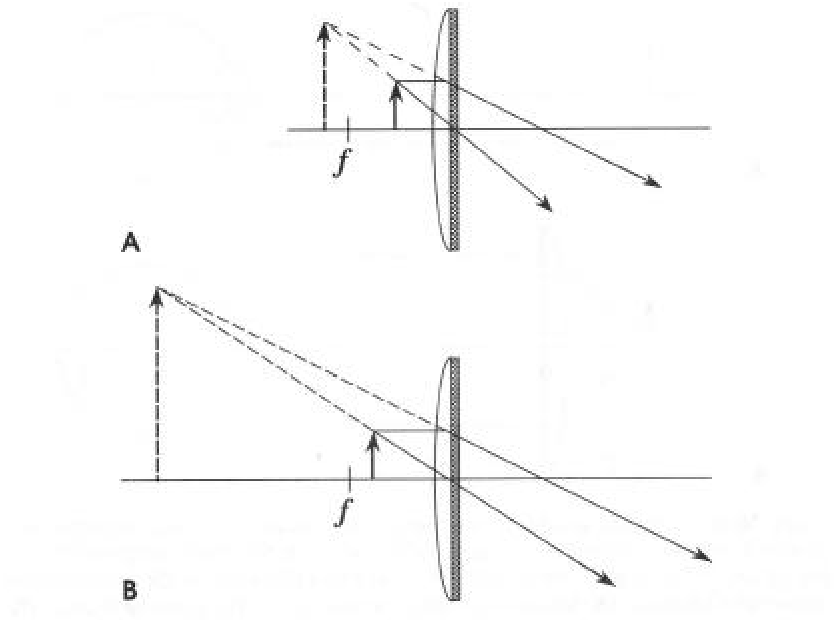


Figure 10–4. (A) A convex lens forms an erect virtual image of an object located between the lens and its anterior focal plane. (B) As the object is moved closer to the anterior focal plane of the lens, the image formed becomes larger and is located farther from the lens.

左の図のように、物体が焦点の内側にある場合は、焦点の外側にある虚像を見ることになる。この場合、虚像までの距離に焦点を合わせるための調節力が必要になる。

スタンドルーペの倍率を知るためには、虚像ができる位置を知らなければならない。

調節力が足りない場合は、Figure10-7 のようにプラスレンズの加入が必要になる。

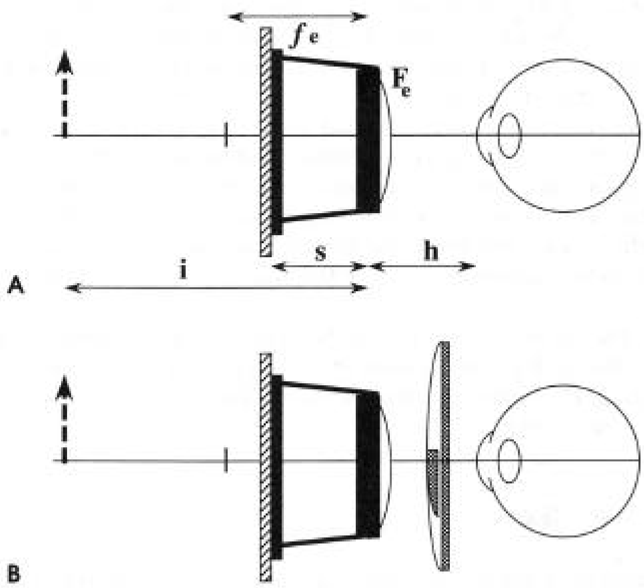


Figure 10–7. (A) A stand magnifier is placed on an object such as an acuity card. The stand height, s, is less than the focal length of the lens, f_e, and therefore the lens forms an erect virtual image (dashed arrow) some finite distance away. (B) Because the image is a finite distance from the eye, a presbyopic person will need an add over best distance correction in order to see it as clearly as possible. The exception to this rule is if the image is actually far enough away that it would be in focus through the distance portion.

スタンドルーペを使用したときの倍率は、次の式により求められる。

M_{total} = RDM × E

RDM: Relative Distance Magnification
= 調節力（cm）／眼からレンズまでの距離（cm）

E：スタンドルーペの実質倍率

■ **Example 10–13.** A stand magnifier enlarges an object 5X, and the image is positioned 20 cm behind the lens. The patient has a +2.50 add and a reference acuity of 0.4/5M. Where must he hold the magnifier, and what size print should he be able to read?

Since the patient is in focus at 40 cm through his bifocal, the image must be located that distance from the spectacle plane. The magnifier lens must therefore be placed 20 cm from the spectacle plane since the image is 20 cm behind it (20 + 20 = 40). The image will be at 40 cm and 5X larger. The total magnification is the product of the RDM and the enlargement, E, by the lens:

$$M_{total} = RDM \times E$$
$$M_{total} = (40/40) \times 5X$$
$$M_{total} = 5X$$
$$\text{Magnification} = \text{Reference size/Goal size}$$
$$5X = 5M/\text{Goal size}$$
$$\text{Goal size} = 5M/5X$$
$$\text{Goal size} = 1M$$

■ **Example 10–14.** A stand magnifier enlarges 3X and has an image location 30 cm behind the lens. The patient is totally presbyopic and has a reference acuity of 0.25/8M through a +4.00 add. She prefers to hold the magnifier 14.5 cm from the spectacle plane. What size print should she read with this stand magnifier, and what add would be needed?

The image will be 44.5 cm from the eye (30 + 14.5 = 44.5), so a +2.25 add is needed (100/44.5 = 2.25).

$$M_{total} = RDM \times E$$
$$M_{total} = (25/44.5) \times 3$$
$$M_{total} = 0.56 \times 3$$
$$M_{total} = 1.68X$$

Magnification = Reference size/Goal size
1.68X = 8M/Goal size
Goal size = 8M/1.68X
Goal size = 4.76M

■ **Example 10–15.** The patient is totally presbyopic and has a best corrected near acuity of 0.4/4M through a +2.50 add. She tries a stand magnifier that has an enlargement of 2.5X and an image location (i) of 20 cm from the lens. How can she use this stand magnifier to read 1M print?

Magnification required = Reference size/Goal size
Magnification required = 4M/1M
Magnification required = 4X

Since the enlargement of the magnifier is only 2.5X, the relative distance magnification must provide 1.6X:

$$M_{total} = E \times RDM$$
$$4 = 2.5 \times RDM$$
$$1.6 = RDM$$

The RDM is found by dividing the reference distance, 40 cm, by the distance of the image from the eye. This latter distance is equal to the image location for the magnifier plus the distance of the magnifier from the eye (h).

$$RDM = 40/(20 + h)$$
$$1.6 = 40/(20 + h)$$
$$h = 5 \text{ cm}$$

E（スタンドルーペの実質倍率）を求めるためには、Fe（equivalent power）および i （レンズから虚像までの距離）を求めなければならない。

Fe（equivalent power）とは、レンズの実質的な度数のことである。光線はレンズを通過するとき、曲面では屈折し、平らな面に垂直にぶつかると屈折せずに直進する。レンズが両凸の場合、レンズメーターでは正確な屈折度数が測定できないと言われている。

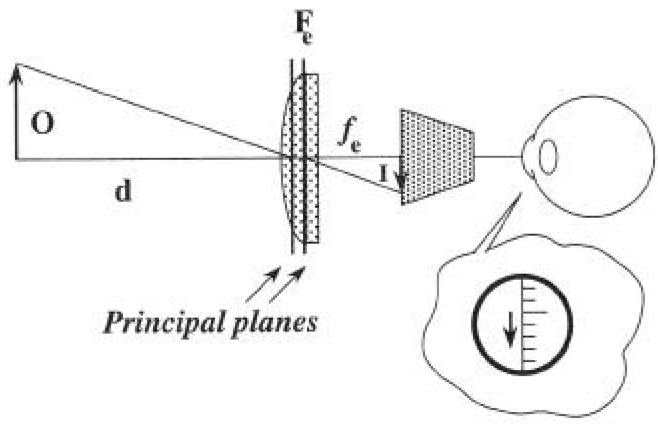


Figure 10–8. The equivalent power of a lens or lens system can be determined by measuring the height (I) of the image formed by an object of known size (O) placed at a known distance (d) from the lens. The image height is measured with a contact lens reticle or similar device. The equivalent focal length, f_e is calculated from similar triangles (f_e = dI/O), and the equivalent power (F_e) is then determined from the relationship $F_e = 1/f_e$.

測定したいレンズを固定し、一定の距離に置いた物体O（倒立像）を見れるようにする。O の大きさおよびdの距離は事前に測定しておく。レンズを通してできる倒立像の大きさ（I）をスケール付のルーペで測定する。

f_e = dI / O　の式の代入し Fe を求める。

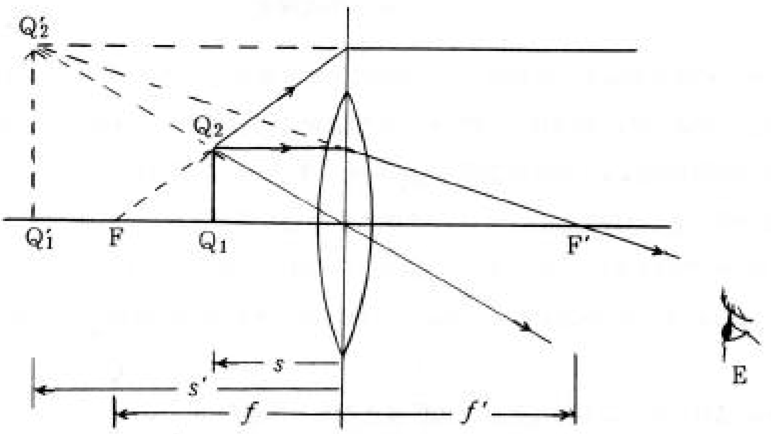


図 4−9　凸レンズによる虚像

$$\frac{1}{s'} - \frac{1}{s} = \frac{1}{f'}$$

左の図で

ｓ：スタンドルーペの脚に相当

ｓ'：レンズから虚像までの距離（I）に相当

ｆ：Ｆｅに相当

これらにより、必要な情報が得られる。

TABLE 10–6. *(continued)*

Magnifier	F_e‡	*E*	*V*§	*i (cm)*	*Reference*
Agfa loupe 8X	28.9	7.80	−4.3	23.5	12
COIL 5123	21.7	6.80	−3.7	26.7	12
COIL 5428	15.5	3.30	−6.8	14.8	12
COIL 5472	5.0	2.00	−5.0	20.0	12
COIL 5474	6.8	2.70	−4.0	25.0	12
Eschenbach 1550	32.5	11.00	−3.2	30.8	12
Eschenbach 1555	20.5	17.40	−1.3	80.0	12
Eschenbach 1558	14.4	3.40	−6.0	16.7	12
Eschenbach 1560	11.6	3.20	−5.3	19.0	12
Eschenbach 1565	5.6	1.80	−7.0	14.3	12
Eschenbach 1580	5.2	1.80	−6.5	15.4	12
Jupiter loupe	12.3	3.90	−4.3	23.5	12
Peak 10X illuminated	22.1	6.9	−3.7	26.7	12
Peak 10X nonilluminated	25.2	7.3	−4.0	25.0	12
Peak 15X illuminated	35.0	8.0	−5.0	20.0	12
Peak 15X nonilluminated	41.0	9.6	−4.8	21.0	12
Selsi Bar #377	25.0	1.50	−50.0	2.0	12
Selsi plano-convex #404 Chrome	10.7	1.60	−17.9	5.6	12
Selsi plano-convex #404 Plastic	10.7	1.80	−13.3	7.5	12

*A value for V was not published and was calculated by the author from i. †A value for i was not published; these values were calculated from V. ‡Calculated from the investigators' data. §F_e and V were not published and were calculated from i and E. Some values in this table may differ slightly from the actual published values due to rounding. It will be noted that the same magnifier may have different values published by different investigators. This is due to variations in measurement as well as variations in quality control by the manufacturers.

TABLE 10–6. Published parameters for selected fixed-focus stand magnifiers.

Magnifier	F_e	E	V	i (cm)	Reference
Agfa-lupe AG-8	29.5	9.00	−3.7	27.1	8
COIL 5123	24.7	7.36	−3.9	25.7	8
COIL 5428	16.1	2.88	−8.6	11.7	8
Jupiter Standlupe #402	12.6	2.98	−6.3	15.8	8
Magnifier	F_e	E	V	i (cm)	Reference
Eschenbach 1153	28.6	26.70	−1.1	91.0	9
Eschenbach 1550	29.8	8.50	−3.7	27.0	9
Eschenbach 1555	24.6	17.40	−1.4	69.0	9
Eschenbach 1558	16.7	4.80	−4.2	24.0	9
Eschenbach 1560	12.9	3.50	−5.0	20.0	9
Eschenbach 1565	7.3	1.80	−9.1	11.0	9
Eschenbach 1580	6.4	1.70	−8.3	12.0	9
Eschenbach 2020	7.7	1.70	−10.0	10.0	9
Eschenbach 2620 High	11.2	2.80	−5.6	18.0	9
Eschenbach 2620 Low	10.8	2.50	−6.3	16.0	9
Eschenbach 2623	6.8	2.10	−6.3	16.0	9
Eschenbach 2625	10.1	2.20	−8.3	12.0	9
Eschenbach 2627	13.4	3.20	−5.6	18.0	9
Magnifier	F_e	E	V*	i (cm)	Reference
COIL 4206	18.2	7.70	−2.7	37.0	10
COIL 4208	22.6	12.50	−2.0	51.0	10
COIL 4210	29.8	10.00	−3.3	30.0	10
COIL 4212	34.6	9.20	−4.2	24.0	10
COIL 5123	23.8	7.00	−4.0	25.0	10
COIL 5226	18.4	8.80	−2.4	42.0	10
COIL 5228	23.2	14.50	−1.7	58.0	10
COIL 5210	29.3	7.20	−4.8	21.0	10
COIL 5212	34.5	7.70	−5.0	20.0	10
COIL 5472	6.6	2.00	−7.1	14.0	10
COIL 5474	8.6	2.50	−5.6	18.0	10
COIL 5428	17.4	3.60	−6.7	15.0	10
Magnifier	F_e	E	V	i (cm)†	Reference
COIL 4206	16.8	7.10	−2.8	36.4	11
COIL 4208	23.0	12.50	−2.0	50.0	11
COIL 4210	27.7	8.80	−3.6	28.2	11
COIL 4212	31.8	8.50	−4.3	23.5	11
COIL 5210	29.2	8.30	−4.0	25.0	11
COIL 5212	34.5	7.90	−5.0	20.0	11
COIL 5213	6.7	2.30	−5.3	19.0	11
COIL 5214	9.7	5.30	−2.3	44.4	11
COIL 5226	16.7	6.60	−3.0	33.3	11
COIL 5228	22.2	11.10	−2.2	45.5	11
COIL 5855	2.3	1.42†	−5.5	18.2	11
Peak 1960	13.9	6.60	−2.5	40.0	11